UNCLASSIFIED

Verification of Constant Absolute Vorticity Trajectories

by

Capt. A. Bruch
U. S. Air Force

Purpose of Investigation

Numerous articles have been written concerning the vorticity equation and the computation of constant absolute vorticity trajectories. The purpose of this investigation is to determine statistically the degree of accuracy of these computed trajectories for twenty-four and forty-eight hour periods, and if possible, to find some systematic departure of the trajectories so that a correction factor might be applied to computations made in the future

Methods

The Bellamy trajectory slide rule was not used for making the computations tested because it is based on the assumption of the earth as a plane. Instead, tables were used which were prepared by interpolation of data taken from curves made by the differential analyzer at the WBAN Analysis Center. These assume a spherical earth.[1]

Several points for computation were selected each day from the 500-mb chart, with the criteria for their selection being the same as established in the theory, i.e., at inflection points or in straight-line flow, in areas of no shear in the horizontal and in areas of no acceleration along the stream-lines.

The wind direction and speed were taken from actual data whenever possible or were computed using the geostrophic assumption. Since non-divergence is another assumption of the conservation of vorticity theorem, the wind was reduced to the level generally considered to be the level of non-divergence, i.e. 600 mb. The method of reduction was to subtract two-thirds of the vertical wind shear between 700 mb and 500 mb from the speed obtained at 500 mb. The wind direction was not changed. The tables were then entered with these values for the wind and the four parameters describing the trajectory were obtained.

The position, wind direction and velocity, and the height of the 500-mb surface at the point selected were recorded, as was the prognostic and verification values of wind direction, and the height of the 500-mb surface. The verification wind direction was taken from the 500-mb contours at the point in question. Separate records were kept of twenty-four and forty-eight hour verifications. Wind speed was not verified. In all, seventy-five trajectories were computed during the period mid-October to mid-December 1951.

1. The personnel involved were changed during the test and were of widely varying experience in the making of these computations.

2. A standard reduction of wind speed from 500 mb to 600 mb must, at times, give erroneous values of wind speed.

---

[1] The tables and instructions are appended.

3. The points were selected from a hemispheric chart with a polar projection. This projection makes the selection of the inflection point extremely difficult.

4. Since the trajectories were primarily for use in a laboratory course, the point of computation was selected such that the verifying wind would be in or near the forecast area in forty-eight hours. This necessitated the initial point of many of the trajectories to be taken in the East Pacific area where analysis is at best not precise.

Examination of results

Table 1 gives the departure, in 10's of degrees, of the observed wind direction from the forecast direction, with positive values indicating an observed wind to the right (clockwise) from the forecast wind. The values for the 24-hour period show a significantly good verification with a reasonably normal distribution and about half of the cases verifying within the accuracy of the wind measurement ($\pm 10^{\circ}$ from zero). The values for 48 hours, on the other hand, do not show a normal distribution and do not show a pronounced maximum near the zero departure. Thus the general conclusion that such trajectories have considerable value for a 24-hour period but little value for a 48-hour period.

Table 2 is the result of the attempt to find some systematic departure of the trajectories. Only 24-hour verifications were used due to the poor distribution of the 48-hour verifications. The various groupings were selected before examination of the data.

The breakdown of initial direction was roughly winds in the southwest quadrant versus those in the northwest quadrant, or, one might say, winds approaching a ridge versus winds approaching a trough. These distributions show no appreciable difference as both approximate a normal distribution. Another choice of longitude intervals would not change the result significantly as the points were quite scattered.

The two wind speed groupings are quite similar and, therefore, they too cannot be used to obtain a systematic departure.

The groupings for the initial longitude, on the other hand, do show some difference. They were chosen so as to approximately separate continental from oceanic areas. This breakdown can be classified generally as the data versus no data excepting the region near the stationary ship in the Gulf of Alaska. As this exception would affect the result only by improving the "no data" distribution, it can readily be said that verification will be better when computations are made either using actual data or having data nearby. Not a startling fact, but one worth considering.

The verifying longitude also shows an interesting difference in the three groups selected. Assuming the potential vorticity to be conserved, winds should deviate to the right of the CAVT when the vertical column is becoming less and vice versa. The grouping of from 125°W to 110°W was selected as that area where the air column would have shrunk due to passage over the mountains. The 105 W° to 100°W group is just east of the mountains where a minimum deviation would be expected and the 95°W to 80°W group is where the column should have stretched enough to show deviation to the left. The figures bear out this selection to some degree although the ±10° deviation still predominates. The few computations at higher latitudes were removed from this sample.

The final classification in Table 2 was selected as it was felt that the change in the height of the particle during the forecast period might indicate energy changes of the particle and thereby give a systematic deviation. This hypothesis again is borne out to some extent by the verification as the interval ±200 ft gives a much more normal distribution than the other grouping..

A final attempt to obtain a prognostically valuable systematic deviation, or rather non-deviation, was to try and find a group of successive days in which the verification was fairly good. The data cards were checked for a string of ±20° verifications and the dates recorded. Three groupings of greater than two days showed up. They were as follows:

1. 22 October through 1 November--17 trajectories 3 exceptions

2. 6 November through 12 November, 10 trajectories 1 exception

3. 22 November through 2 December, 12 trajectories 1 exception

The exceptions are verifications slightly larger than ±20°. These dates were bracketed on a time section of the zonal profile, as computed at Andrews Field and sent out on the circuit "0" teletype. Three day overlapping totals were used to smooth out irregularities in the daily profile. It was then discovered that each of the brackets was centered on an NII stage of the index cycle (1). This supports the notion that vorticity is conserved to a great extend during high index for this index always occurs during an NII stage.

<u>Summary</u>

In spite of the factors noted which might detract from the results, the following was shown more or less significantly concerning constant vorticity trajectories computed under the assumption of a spherical earth:

1. They do not verify well after a 48-hour period.

2. They will verify well after a 24-hour period.

    a. They worked as well anywhere within the range of speeds and direction tested (30 to 80 knots and 170° to 360°).

    b. The verification was somewhat better for starting points over the North American continent than for points over the ocean (probably due to better data and analysis).

    c. The systematic deviation tends to be to the right over mountain areas and to the left some distance to the lee.

    d. Successive days with good verification tend to occur during the NII stage of the index cycle.

(1) Riehl et al, 1952. Forecasting in Middle Latitudes, A.M.S. Monograph No. 5.

CAVT Tables

The data contained in these tables were extracted graphically from CAVT trajectories made by the differential analyzer at the WBAN Analysis Center. The trajectories used were for latitudes 40°, 50°, 60° and 70°N, wind directions of 250°, 210° and 170°, and wind speeds of 30, 50 and 70 knots, making 36 trajectories in all. To obtain the tables, a linear interpolation was assumed to be valid. For this reason, these tables do not pretend to be strictly correct. They are, however, a refinement of the Bellamy Slide Rule trajectories, since the earth's curvature is considered in this method.

The tables require the same parameters as the Bellamy Slide Rule; that is, latitude, wind direction, and velocity. The values then obtained are as follows:

D_U = Distance in degrees longitude between the inflection points of the upper loop.

Y_U = Amplitude in degrees latitude of the upper loop.

D_L = Distance in degrees longitude between the inflection points of the lower loop.

Y_L = Amplitude in degrees latitude of the lower loop.

To obtain the position of the particle at some future time, it is necessary to multiply the wind speed by the time interval desired and lay this distance out along the trajectory.

Note: For wind directions 250° through 170° counterclockwise, draw arrow through origin and measure angle counterclockwise from 90°, e.g. for wind direction 230° enter table using $\alpha = 40°$. For wind directions 290° through 010° clockwise, draw arrow through origin and measure angle clockwise from 90°. Then plot lower loop of trajectory first, e.g. for wind direction 310° enter the table using $\alpha = 40°$.

Table for Computing CAVT's

		D_U					Y_U					D_L					Y_L				
ϕ	α\V→	30	40	50	60	70	30	40	50	60	70	30	40	50	60	70	30	40	50	60	70
	20	30	32	35	36	38	2	2	2	3	3	35	41	48	52	57	4	5	6	6	7
	40	30	33	37	40	42	6	6	6	7	8	31	35	39	42	46	6	7	8	9	10
40	60	30	34	39	43	47	9	10	10	11	12	27	28	30	32	35	9	10	11	12	13
	80	24	28	32	36	39	11	13	14	15	17	18	20	22	23	24	11	12	14	15	16
	100	18	21	25	28	32	14	16	18	19	21	10	12	15	15	16	12	14	16	17	19
	20	32	35	37	39	40	2	2	2	3	3	41	48	55	60	65	4	5	6	7	8
	40	33	37	41	44	48	5	6	7	7	8	35	40	44	48	52	7	8	9	10	11
45	60	34	39	44	49	54	9	10	11	12	13	28	31	34	36	39	9	11	12	14	15
	80	28	33	37	42	47	12	13	15	16	17	19	22	24	26	28	11	13	15	16	17
	100	22	26	30	35	39	14	16	18	20	21	11	13	15	17	19	13	15	17	19	20
	20	35	37	40	41	43	2	2	2	2	2	48	55	62	67	73	5	6	7	8	9
	40	37	41	45	49	53	5	6	7	7	8	39	44	50	54	58	7	9	10	11	13
50	60	39	44	50	56	62	8	10	11	12	13	30	34	38	40	43	10	12	13	15	16
	80	32	38	43	49	54	12	13	15	16	17	21	24	26	29	32	12	14	16	17	19
	100	26	31	36	41	47	16	17	19	20	22	12	13	16	18	22	14	16	18	20	21
	20	39	41	43	45	46	2	2	2	2	2	57	66	75	81	87	6	7	8	9	10
	40	43	48	52	56	59	5	6	7	7	7	46	53	59	64	69	8	10	11	12	14
55	60	47	54	61	66	72	9	10	11	12	12	35	39	43	47	50	10	12	14	16	17
	80	40	47	54	61	68	12	14	15	16	17	24	27	30	33	36	12	14	16	18	20
	100	33	40	48	56	63	15	17	19	21	22	14	16	18	20	22	14	16	18	20	[illegible]
	20	43	45	47	48	49	2	2	2	2	2	67	77	88	95	102	6	8	9	10	11
	40	49	54	59	62	65	5	6	6	7	7	53	61	[illegible]	74	80	9	10	12	13	14
60	60	56	64	72	77	82	9	10	11	11	12	40	44	48	53	58	11	13	15	16	18
	80	48	57	66	73	81	12	14	15	16	17	28	31	34	37	40	13	15	17	19	20
	100	40	50	60	70	80	16	18	19	21	22	16	18	21	22	23	15	17	19	21	23
	20	47	48	49	50	51	2	2	2	2	2	89	100	113	118	125	7	9	10	12	13
	40	60	60	70	73	75	6	6	6	6	6	70	78	85	91	96	10	11	13	15	16
65	60	73	82	91	95	99	9	9	10	11	11	51	56	61	64	68	12	14	16	18	19
	80											33	38	42	44	47	14	16	18	20	22
												16	19	22	24	26	17	19	21	23	24
	20	51	51	52	52	53	1	1	1	1	1	112	123	134	141	148	8	10	12	13	15
	40	71	76	81	83	84	5	5	6	6	6	87	95	104	108	113	11	13	14	16	18
70	60	91	100	110	113	116	8	9	9	10	10	62	68	74	76	78	13	15	17	19	21
	80											39	44	49	51	53	16	18	20	22	23
	100											16	20	24	26	29	19	21	23	24	25

Time Interval	Departure in degrees (+ = departure to right)						
	<-50	-40 and -50	-20 and -30	-10 0 +10	+20 and +30	+40 and +50	>+50
24 hour	7	7	17	47	13	3	7
48 hour	19	7	19	24	9	8	15

Table 1--Departure of Observed Wind Direction from Forecast Wind Direction (as per cent of total)

Parameter		Departure in degrees (+ = departure to right)				
		<-30	-20 and -30	-10 0 +10	+20 and +30	>+30
Initial Direction	170° thru 260°	9	13	27	5	5
	270° thru 360°	4	4	20	8	4
Initial Speed (Knots)	30-50	8	13	33	5	8
	55-80	5	4	15	7	1
Initial Longitude (°West)	75° thru 125°	3	8	27	5	3
	130° thru 180°	9	10	20	8	8
Verifying Long. (°West)	125-110	1		12	8	
	105-100	1		13	4	
	95-80	9		9	4	
24 hr. Height Change (Initial-Final)	+200 thru -200	5	9	28	5	3
	>+200 and <-200	7	8	19	8	8

Table 2--24 hour Departure of Observed from Forecast Wind Direction vs. Various Parameters (as per cent of total)